PRODUCTION RUN LENGTH MODEL CONSIDERING INSPECTION AND PREVENTIVE MAINTENANCE

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Abstract

This research deals with a production run length model for a deteriorated production system, and considers process controls in term of inspection, preventive maintenance and restoration. Deterioration causes the shifting of the system state randomly, from in control state to out of control state. During out of control state, the system may produces non conforming products. Because of this condition, the system needs inspection to check the system state and preventive maintenance to avoid an excessive deterioration. Preventive maintenance can reduces the failure rate of system. If inspection and preventive maintenance is done too often, the system will be more reliable but costs of inspection and preventive maintenance will increase. Determination an appropriate interval inspection balances the trade off between inspection and preventive maintenance cost versus non conforming products cost. This research determinates a production run length and number of inspection during that production run length. From numerical example, it can be concluded that the minimum expected total cost per unit of production systems which apply preventive maintenance is less or equal than those which do not apply preventive maintenance.

Keywords: *Production run length, deterioration, IFR, periodic inspection, imperfect maintenance*

INTRODUCTION

In general, a mass production system will be deteriorated due to usage and or age. The deterioration can causes a system shifts from in control state to out of control state. During out of control state, a system has more probabilities to produce non conforming items. First effort to ensure a production system be in control state is doing an inspection. An inspection is an activity to check system conditions or states. While during inspection the system is found out of control, an action will be taken to return the system to in control state. This action usually called restoration. In a deteriorated production system, the shifting of states form in control to out of control will faster and can be modelled by increasing failure rate. Often an inspection is not enough to control the production system. Preventive maintenance is needed to decrease the deterioration.

In many researchs, a mass production system is modelled by Economic Manufacturing Quantity (EMQ). Researchs which study a deteriorated EMQ are Rosenblatt and Lee (1986) and Porteus (1986). The researchs showed the optimal production run length is shorter than the classical EMQ. Then Lee and Rosenblatt (1987) developed an EMQ model which considering inspections. The solutions yielded bigger lot sizes by doing inspections. Kim and Hong (1999) extended Lee and Roseblatt (1987) by assumed that time of system shifts to out of control has general distributions. Wang and Sheu (2001) and Wang (2005) did the DTFs (disregard the firsts) inspection policy. Tseng (1996), Tseng et al (1998), Ben Daya (2002), Wang (2006) studied EMQ which considering inspections and preventive maintenances to control processes. Tseng (1996) did perfect maintenance after periodical inspections. Tseng (1998) continued Tseng (1996) by assumed the maintenance is imperfect, that is maintenance can bring processes from in control state to out of control. Ben-Daya (2002) applied imperfect preventive maintenance which reducing the age of system. But if during inspection system is found out of control, the the production cycle is stopped.

In conditions where the system can restored and maintained without stopping whole system, then restoration and maintenance can be done without stopping the production cycle. Also stopping production cycle will complicates the administration work.

For example of this system is production system which uses chemical milling machine.

This paper studies determination of production run length dan number of inspections in a deteriorated production system which has increasing failure rate and conducts periodical inspections and preventive maintenance. Time to conducts inspection, maintenance and restoration is assumed very less so it negligible.

The outline of this paper is as follows. In section 2 we provide the notations to formulate the model. In section 3 we explain steps to formulate the model. In section 4 we analyze the model and method to find solution. In section 5 we give a numerical example to describe the model. Finally in section 6 we gives conclusions and discussions for future work.

NOTATIONS

The following notations are used to formulate the model:

d demand rate
p production rate
k set up cost
h holding cost
v inspection cost
s non conforming product cost
r restoration cost
α proportion of non conforming products during uncontrol state
ϕ fixed cost of preventive maintenance
γ variable cost of preventive maintenance, influenced by δ
Q lot size
τ inspection interval length
τ_i i^{th} inspection time
δ_i the reduction of failure rate after preventive maintenance at τ_i
z_j time of first system shifts to out of control state after restoration conducted at τ_j
$h_0(t)$ beginning failure rate
$h_{ij}(t)$ failure rate during (τ_{i-1}, τ_i) after restoration conducted at τ_j
A_i probability of system in out of control state at τ_i
$F_0(t)$ beginning *cdf* of system
$F_{ij}(t)$ *cdf* of system during (τ_{i-1}, τ_i) if restoration conducted at τ_j
$f_{ij}(z)$ *pdf* of z if out of control state is found during (τ_{i-1}, τ_i)
$S(T)$ set up cost per unit
$H(T)$ holding cost per unit
$I(T,n)$ inventory cost per unit
$R(T,n)$ restoration cost per unit
$P(T,n)$ preventive maintenance cost per unit
$C(t,n)$ non conforming product cost per unit
$TC(T,n)$ total cost per unit
T production run length
n number of inspections

MODEL FORMULATION

System Description

We consider a single unit mass production system which deteriorated with increasing failure rate dan has no machines stop. In beginning production cycle, system always in control state. During in control state, system produces conforming products. After some production times, system shifts to out of control state. During out of control state, system produces non conforming products as many as α percent.

For checking system state, an inspection is conducted during production run length $(0, T)$ at time $\tau_i = i\tau$, $i = 1, 2, ..., n$, $\tau_n = n\tau = T$. If the inspection finds that system is in out of control state, then restoration will conducted. But if the system is in control state, then only preventive maintenance will conducted.

Restoration will return the system from out of control state to new condition. The restoration is conducted at τ_n and if system is found out of control during inspection.

Preventive maintenance will improve system by decreasing failure rate as many as δ_i at maintenance time τ_i. This preventive maintenance is modelled by preventive maintenance model used in Jack and Murthy (2002).

The relation between failure rate and the inspection-preventive maintenance policy is described in Figure 1.

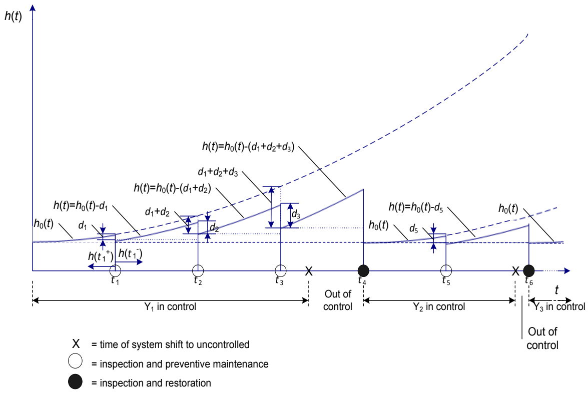

Figure 1
The relation between failure rate and time if system conducts inspection-maintenance-restoration policy

By conducting this policy, expected total cost per cycle will include set up cost, holding cost, inspection cost, restoration cost, preventive maintenance cost and non conforming product cost. The policy is aimed to determine production run length T^* and inspection number n^*, so that expected total cost per unit is minimized.

Model Formulation

We consider costs as follow:

a). Set up cost, $S(T)$

Set up is conducted once at the beginning cycle.

So set up cost per unit product is: $S(T) = \frac{k}{pT}$

b). Holding cost, $H(T)$; $H(T) = \frac{(p-d)Th}{2d}$

c). Inspection cost, $I(T,n)$

During one production cycle, inspection is conducted n times. So inspection cost per unit is:

$$I(T,n) = \frac{nv}{pT}$$

d). Expected restoration cost, $R(T,n)$

Restoration is conducted at the end of production cycle τ_n and at times τ_i, $i = 1, 2, ..., n-1$ if during inspection system is found out of control. If A_i is the probability that system will found out of control at τ_i so expected restoration per production cycle is $\left\{1 + \sum_{i=1}^{n-1} A_i\right\}$.

Let $F_{ij}(t)$ denotes $F(t)$ during interval (τ_{i-1}, τ_i) if last restoration was conducted at τ_j, then

$$F(\tau_{i-1},\tau_i) = \begin{cases} 0 & for\ \ i = 1 \\ \sum_{j=0}^{i-2} A_j\left[F_{ij}(\tau_i - \tau_j) - F_{ij}(\tau_{i-1} - \tau_j)\right] & for\ \ i \geq 2 \end{cases}$$

A_0=1 and let $F_{ij}(\tau)$ denotes $F_{ij}(\tau_i\text{-}\tau_j)\text{-}F_{ij}(\tau_{i-1}\text{-}\tau_j)$ then A_i is formulated as:

$$A_i = \begin{cases} F_0(\tau) & for\ \ i = 1 \\ A_{i-1}F_0(\tau) + (1 - A_{i-1})\left\{\sum_{j=0}^{i-2} A_j\left[F_{ij}(\tau)\right]\right\} & for\ \ i \geq 2 \end{cases}$$

So expected restoration cost per unit product is given by:

$$R(T,n) = \begin{cases} \frac{r}{pT} & for\ \ n = 1 \\ \frac{r\left\{1 + \sum_{i=1}^{n-1} A_{i-1}F_0(\tau) + (1 - A_{i-1})\left(\sum_{j=0}^{i-2} A_j F_{ij}(\tau)\right)\right\}}{pT} & for\ \ n \geq 2 \end{cases}$$

e). Expected maintenance cost, $P(T,n)$

Preventive maintenance will conducted if during inspection system is found in control. For optimizing preventive maintenance, the maintenance is done by decreasing failure rate as many as posible (Jack and Murthy, 2002), so $\delta_i = h_0(\tau_i) - h_0(\tau_{i-1})$.

If preventive maintenance cost at τ_i is $\phi + \gamma\delta_i$, then expected maintenance cost at interval (τ_{i-1}, τ_i) is $(\phi + \gamma\delta_i)$ $(1 - A_i)$. So expected preventive maintenance cost is:

$$P(T,n) = \begin{cases} 0 & for\ \ n = 1 \\ \frac{\{\phi + \gamma[h_0(\tau_i) - h_0(\tau_{i-1})]\} \times \sum_{i=i}^{n-1}\left\{1 - \left[A_{i-1}F_0(\tau) + (1 - A_{i-1})\left(\sum_{j=0}^{i-2} A_j F_{ij}(\tau)\right)\right]\right\}}{pT} & for\ \ n \geq 2 \end{cases}$$

f). Expected non conforming product cost, $C(T,n)$

Non conforming product cost is occured because during out of control state, system produces non conforming product as many as α percent.

If z denotes random variable of first time system shifts to out of control state after restoration at τ_j, $\tau_{i-1}\text{–}\tau_j \leq z \leq \tau_i\text{–}\tau_j$ and $f_{ij}(z)$ denotes *pdf* of z then expected number of non conforming products during interval (τ_{i-1}, τ_i) is given by:

$$\begin{cases} \alpha p\left\{\int_0^{\tau}(\tau - z)f_{ij}(z)dz\right\} & for\ \ i = 1 \\ \alpha p\sum_{g=0}^{i-1}\left[A_{g-1}F_0(\tau) + (1 - A_{g-1})\left(\sum_{j=0}^{g-2} A_j\left[F_{gj}(\tau)\right]\right)\right]\left\{\int_{\tau_{i-1}-\tau_g}^{\tau_i - \tau_g}(\tau_i - \tau_g - z)f_{ij}(z)dz\right\} & for\ \ i \geq 2 \end{cases}$$

So expected non conforming product per unit is given by :

$$C(T,n) = \frac{s}{pT}\begin{cases} \alpha p\left\{\int_0^{\tau}(\tau - z)f_{ij}(z)dz\right\} & for\ i = 1 \\ \alpha p\left[\int_0^{\tau}(\tau - z)f_{10}(z)dz\right] + \sum_{i=2}^{n}\alpha p\sum_{g=0}^{i-1}\left[A_{g-1}F_0(\tau) + (1 - A_{g-1})\left(\sum_{j=0}^{g-2} A_j\left[F_{gj}(\tau)\right]\right)\right]\left[\int_{\tau_{i-1}-\tau_g}^{\tau_i - \tau_g}(\tau_i - \tau_g - z)f_{ij}(z)dz\right] & for\ i \geq 2 \end{cases}$$

Expected total cost per unit, $TC(T,n)$

$TC(T,n)= S(T)+H(T)+I(T,n)+R(T,n)+P(T,n)+C(T,n)$

Subject to $T > 0$

$n \geq 1; \quad n \in \{I\}$

MODEL ANALYSIS

The expected total cost per unit involves variables T and n, where n is integer. Analytical analysis of T and n simultaneously will so hard, then optimal solution is obtained using numerical method.

NUMERICAL EXAMPLE

We consider that failure rate of production system follows Weibull distribution with scale parameter $\lambda = 1$ dan $\beta = 2{,}5$. We assume that parameter values are $p = 750$, $d = 500$, $\alpha = 0.2$, $h = 0.5$, $k = 150$, $v = 10$, $r = 100$, $s = 20$, $\phi = 10$, $\gamma = 10$.
Numerical solution is obtained by using matcad 13.

In this section, we first show optimal production run length T^* and its expected total cost $TC(T^*, n)$ for number of inspection n 1 to 8 in system which conducts preventive maintenance and system without preventive maintenance. (see Table 1)

Table 1 Optimal $TC(T^*,n)$ in system with and without maintenance

N	With maintenance		Without maintenance	
	T^*	$TC(T^*\|n)$	T^*	$TC(T^*\|n)$
1	0.55	20.940	0,55	20,940
2	0.76	20.770	0,63	20,913
3	0.91	20.712	0,71	20,898
4	1.025	20.688	**0,78**	**20,895***
5	1.12	20.680	0,85	20,898
6	**1.19**	**20.679***	0.91	20.905
7	1.26	20.683	0.97	20.914
8	1.31	20.690	1.02	20.926

In Table 1 we find that preventive maintenance can reduces the optimal expected total cost per unit, as with maintenance total cost is 20.679 and without maintenance is 20.895. Also we find that preventive maintence influences the optimal production run length, because during inspection we also conduct preventive maintenance to increase system reliability.

And then we studied sensitivity analysis to find the influence of model parameters to model solution. Sensitivity analysis is done by changing ratio of maintenance cost toward to non conforming products cost and percentage of non conforming products. (see Table 2 and Table 3)

Table 2 The effect of ratio of maintenance and non conforming product cost toward to n,T, $TC(T,n)$

Rasio	n^*	T^*	t	$TC^*(T,n)$
0,25	7	1,21	0,172	20,624
0,5	6	1,17	0,195	20,654
0,75	**6**	**1,19**	**0,198**	**20,679**
1	5	1,13	0,226	20,703
1,25	5	1,14	0,228	20,725

Table 3 The effect of percentage of non conforming product toward to n,T, $TC(T,n)$

n^*	T^*	$TC^*(T,n)$
4	1,24	20,630
6	**1,19**	**20,679**
7	1,12	20,737
8	1,10	20,777

Table 2 shows that preventive maintenance cost will influence the model in term of reducing number of inspection. If preventive maintenance cost is very expensive, then the model tend to chooses inspection number as 1 or system will always conduct restoration after inspection. Table 3 shows that percentage of non conforming product will push the system to conduct more inspections.

CONCLUSIONS

In this paper we studied model to determine production run length in deteriorated production system which considering process control as inspection, preventive maintenance and restoration. The model gives production run length and inspection number which minimize expected total cost per unit. From numerical example we obtain that expected total cost per unit will less if system conducts preventive maintenance. The model assumes there's no inspection mistakes and products sold without warranty, model which considering inspection mistakes and or warranty can be developed for the future work.

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