ENTROPY INCREASE RELATIONS IN RANKINE CYCLE'S WORK PROCESSES

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Abstrak

Hubungan antara kenaikan entropi dan efisiensi total untuk dua peralatan yang terkait dengan langkah kerja pada suatu siklu suap (Rankine) memiliki arti yang signifikan. Hubungan ini diperlukan untuk menentukan destruksi kerja berguna. Pada siklus tersebut sebagai salah satu langkah dalam melakukan evaluasi Hukum Termodinamika Kedua atau dalam melakukan studi Minimisasi Pembangkitan Entropi *(Entropy Generation Minimization)*. Tulisan ini akan menguraikan dan mendiskusikan hubungan tersebut.

Kata Kunci: siklus, Rankine, entropi

BACKGROUND

In any steam cycle, particularly for Rankine cycle, there are at least two work-related processes. They are compression-work process (by a feed-water pump) and expansion-work process (by a steam turbine) as given by Fig. 1. The two work-related processes are usually idealized as isentropes. In order to accommodate the fact that practically the two processes deviate from isentropic prompted the introduction of the so-called isentropic efficiencies describing the ratio between ideal work (when the device operates isentropically) and actual work for compression being performed by boiler feed-water pump. On the opposite, the efficiency of expansion process performed by steam turbine the orders of denominator and numerator are reversed, i.e., the ratio between actual and ideal work. It is understood here that these two works are those works that are imposed by the equipment (pump or turbine) to the working fluid.

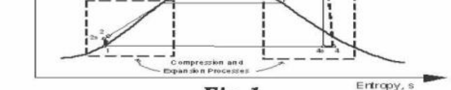

Fig. 1.
Simple Rankine cycle depicting compression and expansion processes

Usually classical textbooks on Thermodynamics stop the discussions at this point whereas the application of the Second Law of Thermodynamics is now covering wider applications and gaining more importance in measuring the goodness of a process or cycle. That is, to measure the effectiveness of a process it is necessary to be able to determine the increase in entropy caused by the inefficiencies of pumps and turbines in their operations. The increase in entropy is closely related to the amount of entropy generation that in turn will determine the rate of irreversibility. The following discussions are aimed at deriving and determining the amount of increase of entropy due to the inefficiencies experienced by pumps and turbines. For steam turbine, the derivations are made following the lead suggested by Cole (1996).

Entropy Increase in Pump Operation

Fig. 1 schematically depicts ideal or isentropic process denoted by the subscript "s" while the actual process is denoted by the subscript "a" for pump works. From standard Thermodynamics textbooks such as Çengel and Boles (1989), the efficiency of pump as a work-consuming device is given by the following expression

$$\eta_{P,s} = \frac{w_{P,s}}{w_{P,a}} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \tag{1}$$

By putting $h_{2a} > h_{2s}$ (so that) $\eta_{P,s} < 100\%$ the following relation can be written

$$h_{2s} - h_{2a} = \Delta h$$
$$h_{2a} = h_{2s} + \Delta h$$

Where is the enthalpy increase of the working fluid due to work dissipation in the form of heat because of fluid friction as one of the losses to be endured by pumps. Based on the enthalpy relation above, equation (1) can then be written as

$$\eta_{P,s} = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{h_{2s} - h_1}{(h_{2s} - h_1) + \Delta h} = \frac{w_{P,s}}{w_{P,s} + \Delta h}$$

from which

$$\Delta h = w_{P,s}\left(\frac{1}{\eta_{P,s}} - 1\right) \qquad (2)$$

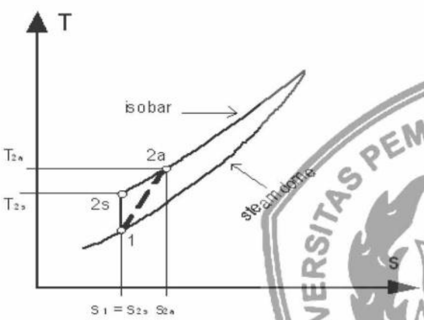

Fig. 2. *Pump operation schematic*

By invoking Maxwell's relation that relates entropy and enthalpy changes in an isobaric process, then

$$Tds = dh - \underbrace{vdP}_{=0} = dh$$

If this last equation is integrated from "2s" as the upper bound and "2a" as the lower bound, the relation below is obtained. However, before integrating the equation, it is necessary to appreciate the fact that the above equation can only be integrated if the relation between entropy and temperature is known. Considering this integration is aimed at discovering the effect of pump efficiency on entropy increase and remembering that the point "2s" is the final state to be reached if the compression process by the pump is done isentropically, then the most plausible decision is to select the temperature to be . This choice confirms the fact that however close the pump efficiency to 100%, this small deviation will, by any means, contribute to the increase of working fluid's entropy. Hence, the election of as reference temperature has strong justification so that the above integration will result in

$$\int_{2s}^{2a} dh = h_{2a} - h_{2s} = \int_{2s}^{2a} Tds = T_{2s}\int_{2s}^{2a} ds = T_{2s}\Delta s$$

By substituting this last relation to equation (2), the following expression will be obtained

$$T_{2s}\Delta s = w_{P,s}\left(\frac{1}{\eta_{P,s}} - 1\right)$$

from which

$$\Delta s = \frac{w_{P,s}}{T_{2s}}\left(\frac{1}{\eta_{P,s}} - 1\right) \qquad (3)$$

Emphasize is in order here that entropy increase given by equation (3) is the increase in entropy for the working fluid because all the derivations so far are focused on the pump's working fluid. Also, the ideal work imparted by a pump to its working fluid is the smallest work needs to be "purchased" to operate it. Mechanics of fluids calls this work as fluid power. In practice, however, there are a number of other works to be provided to operate a pump, for example, works to overcome friction, flow leakage (which is related to volumetric efficiency), and other works for the same purposes. Together, these individual "purchased energies" in the form of losses need to be added up obtain all the fluid flow and mechanical energy consumptions to operate a pump.

Referring to Church (1993), it is stated that pump's total work is obtained by summing up a number of works needed to overcome resistances hampering the pump from performing its duty. In other words, total efficiency of a pump is arrived at by multiplying a number of relevant 'local' efficiencies such as mechanical efficiency, hydraulic efficiency, volumetric efficiency, etc. which may differ from one pump manufacturer to another either in terminology or in concept. But all these local efficiencies can be combined into overall or total efficiency that may be written as

$$\eta_{P,tot} = \eta_{P,s} \times \eta_m \times \cdots \times \eta_v$$

The total entropy increase caused by pump operation henceforth becomes

$$\Delta s = \frac{w_{P,s}}{T_{2s}}\left(\frac{1}{\eta_{P,tot}} - 1\right) \quad (4)$$

Equation (4) indicates that the ratio outside the brackets will always yield a constant value for a fixed set of operating conditions so that the magnitude of entropy increase will be directly proportional to the inverse of pump's total efficiency. The lower the total efficiency, the higher the increase of entropy will be. Logically, with the increase in entropy getting higher, the rate of irreversibility will also increase asymptotically. In turn, the increase of irreversibility is a manifestation of destruction of useful work that as far as possible must be minimized.

Equation (4) can be cast in a more useful dimensionless number. Tentatively, the dimensionless number is called Pump Entropy Number, , and is defined as

$$N_{\Delta s,P} = \frac{T_{2s}\,\Delta s}{w_{P,s}} = \left(\frac{1}{\eta_{P,tot}} - 1\right) \quad (5)$$

It is interesting to examine this last equation with the help of Fig. 3 given below. Equation (5) above indicates that pump entropy number will reach infinity as In order to appreciate the magnitude of this number, on the upper left-hand side of Fig. 3, the pump entropy number is depicted by varying pump efficiency's from 0% to 1%. It is seen that the number can reach as high as 9,999 which is a fairly big number for efficiency of 0.01%. Also, the change in pump entropy number as the pump efficiency varies from 50% to 100% is given on the right-hand side of Fig. 3.

This graphical presentation will immediately raise the question: why is there no definite upper bound for the pump entropy number whereas the lower bound can be exactly located? The answer is (compare also with turbine entropy number to be derived below) that we can supply unlimited amount of energy to force the pump to work as a work-consuming device but if it has refused to work (for example, because the pump is jammed) then all the energy supplied will contribute only to the destruction of useful work. It is necessary to emphasize here that Fig. 3 assumes the pump is constructed with infinite strength. In reality, if the energy supplied to this jammed pump has become enormously high then it might be possible that all this energy is used up to smash the pump's impellers into pieces. When this situation is reached, the pump becomes a completely useless piece of equipment and no matter how big the energy is supplied to this "broken pump", it will never want nor will it do its intended duty. This phenomenon is common to all work-consuming devices.

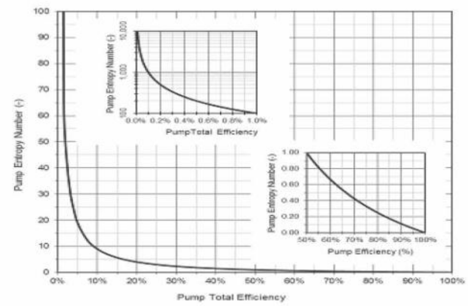

***Fig. 3** Pump Entropy number*

Entropy Increase in Steam Turbine Operation

Fig. 4 below presents schematically the ideal (isentropic) expansion work performed by steam turbine that is denoted by subscript "s" and the actual expansion process denoted by subscript "a." Once again, standard Thermodynamics textbooks such as the one by Çengeldan Boles (1989) give the following formula to determine isentropic efficiency of a steam turbine as a work-producing device

$$\eta_{T,s} = \frac{w_{T,a}}{w_{T,s}} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \quad (6)$$

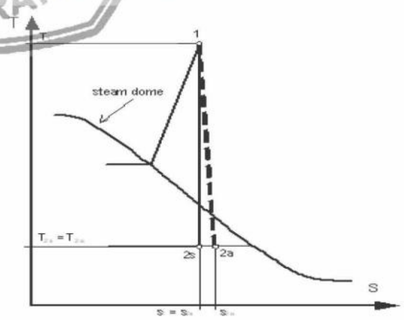

***Fig. 4.** Steam turbine operation schematic*

From the same source, in the discussion of phase equilibrium, it is said that the condition for equilibrium to take place under the steam dome (where liquid and vapor co-exist) is when the Gibbs function attains the same value everywhere in that region, Fig. 4. This condition is quoted here because generally the end state of a condensing turbine expansion (point 2 in Fig. 4) is inside the two-phase region for which Nag (2001) points out that the steam quality is usually about 90%. As is known, specific Gibbs function is defined as such that for isentropic (ideal) and actual expansions performed by a steam turbine it can be written as

$$g_1 - g_{2a} = (h_1 - T_1 s_1) - (h_{2a} - T_{2a} s_{2a}) \quad (7)$$

for actual expansion and

$$= (h_1 - h_{2a}) - (T_1 s_1 - T_{2a} s_{2a})$$
$$= \eta_{T,s} w_{T,s} - (T_1 s_1 - T_{2a} s_{2a})$$

$$g_1 - g_{2s} = (h_1 - T_1 s_1) - (h_{2s} - T_{2s} s_{2s}) \quad (8)$$
$$= (h_1 - h_{2s}) - (T_1 s_1 - T_{2s} s_{2s})$$
$$= w_{T,s} - (T_1 s_1 - T_{2s} s_{2s})$$

for isentropic expansion.

Invoking phase-equilibrium condition stating that makes equation (7) = equation (8). In addition to the fact that under the steam dome, the following expression can be deducted

$$\eta_{T,s} w_{T,s} - (T_1 s_1 - T_{2a} s_{2a}) = w_{T,s} - (T_1 s_1 - T_{2s} s_{2s})$$
$$T_{2s}(s_{2a} - s_{2s}) = w_{T,s}(1 - \eta_{T,s})$$

that leads to

$$\Delta s = s_{2a} - s_{2s} = \frac{w_{T,s}}{T_{2s}}(1 - \eta_{T,s}) \quad (9)$$

Again, it needs to be emphasized the entropy increase formulation according to equation (9) is for the working fluid because all the derivations so far have been focused on the working fluid. In addition, the ideal work produced by a steam turbine by expanding the working fluid is the highest work that can be harvested from operating it. However, in practice, there are a number of work expenditures that must be spent to operate a steam turbine, for example, the works expended to overcome friction, flow leakages, and a number of other works. In turn, these work expenditures will decrease the net work that can be extracted from turbine's expansion.

According to Saarlas (1978), total turbine efficiency is obtained by subtracting a number of works needed to overcome several resistances in performing its function. In other words, total efficiency for a steam turbine is obtained by multiplying several relevant 'local' efficiencies such as mechanical efficiency and leaving-loss efficiency that may be different from one manufacturer to the other either in its naming convention or concept. But all these 'local' efficiencies may be grouped into total efficiency that takes the form of

$$\eta_{T,tot} = \eta_{T,s} \times \eta_m \times \cdots \times \eta_l$$

Thus the overall or total increase in entropy in operating a steam turbine becomes

$$\Delta s = \frac{w_{T,s}}{T_{2s}}(1 - \eta_{T,tot}) \quad (10)$$

Similar to that for pump, equation (4), it is seen that the division outside the bracket will always give a constant value so that the increase in entropy for turbine is proportional to the subtraction of turbine's total efficiency from unity. The lower the turbine's total efficiency, the higher the increase of entropy will be. In turn, the higher entropy increase will accordingly cause the rate of destruction of useful work to increase as well. To make the steam turbine operates more efficiently; it is imperative to minimize the amount of useful work destruction.

Following equation (5) above, equation (10) may be rearranged in the form of a non-dimensional number that will give deeper insights to the amount of entropy increase. Tentatively, this dimensionless number is called Turbine Entropy Number, , which is defined as

$$N_{\Delta s,T} = \frac{T_{2s}\Delta s}{w_{T,s}} = (1 - \eta_{T,tot}) \quad (11)$$

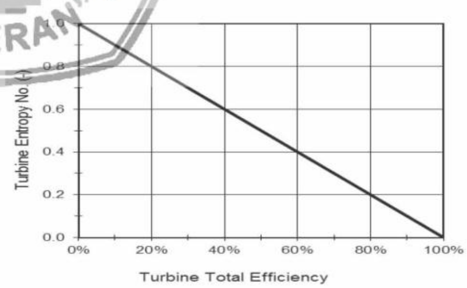

Fig. 5. *Turbine entropy number*

This last equation together with Fig. 5 given in the end of this paper is interesting to examine. As is indicated by equation (11), turbine entropy number formulation gives a straight line with negative 45° slope. It is also seen that turbine entropy number takes the form of a simple relation with turbine's efficiency in which the terminal points of turbine *entropy number can be a priori specified, that is, the terminal points are either unity or zero. This is because when the efficiency equals 0% there will be no work produced irrespective of how hard the turbine being forced or persuaded to function. Thus the case of turbine is diametrically different with that of pump as discussed above and, once again, this phenomenon characterizes all work-producing devices.*

It is relevant to elaborate that the linear relation for steam turbine and the more complex relation for pump above are based on the differing roles between the two; one is work-producing device while the other is work-consuming device. The addition of 'local efficiencies' for each device increases the complexity of energy expenditures within the devices. Hence, it is a constant challenge for researchers to clearly and definitely divide those energy expenditures according to the phenomena causing them and strive to minimize them as appropriate technologies become available. Indeed, it is not a very difficult exercise to confirm such efficiencies. Take a pump as a common device that finds wide use even in household; for example, it is noticed that its total efficiency varies with increasing or decreasing rate of flow. The same applies to a steam turbine so that if the non-constant total efficiencies for turbine are substituted into equation (11), a non-linear relation will be resulted. Complexity of turbine and pump entropy numbers are very much dependent on the operational parameters of each device which usually provided by its respective manufacturer.

APPLICATIONS

The derivations above do not only enrich the understanding of how work-related devices consume or produce 'driving power' in order to operate but can also be applied to measure such consumption. For pump, as the first of such devices, practical on-site measurements can be performed as the basis to calculate operational efficiency that may be different from the value specified by manufacturer because of the efficiency calculated as such is free from any assumptions. To conduct properties measurements of the working fluid, at the minimum, two pressure gauges must be available, one for suction side and the other for discharge, one temperature indicator and one flow meter. Preferably, temperature indicator and flow meter are installed at the same side. Electrical measurements can be made using clamp-on meter to determine the actual total work supplied while non-intrusive torque meter to measure shaft work. The data thus collected can be utilized to calculate isentropic efficiency and total actual efficiency. Nevertheless, the other individual (or 'local') efficiencies mentioned above can only be lumped in to an aggregate after dividing the actual total efficiency by the isentropic efficiency. The same measurement configuration can be applied to steam turbine but with a slight inaccuracy because in reality the process under the steam dome for Rankine cycle is usually slanted slightly towards lower pressure as the steam condenses, hence, is not exactly equal to for steam turbine due to the fact that the two points now are not on the same straight line. The efficiency aggregate above has more components if both the pump and turbine under observation are multi-stage ones. Having the values of Pump or Turbine Entropy Number, entropy increase, ,can be calculated using either equation (4) or (10). Based on these the irreversibility rate of each device can be estimated using the relation

$$\dot{I} = \dot{m}T_0\Delta s \qquad (12)$$

The irreversibility can be seen as the "fuel" to drive the pump and turbine for as is known the First Law of Thermodynamics, also known as the Law of Energy Conservation states that energy cannot be created nor destroyed, hence, the Second Law of Thermodynamics is the one law that can indicate what kind of "fuel" spending must be expended to operate a pump or turbine. The worse the design of the pump and or turbine, the greater the required "fuel" to drive the machinery will be. This "fuel" is basically the destruction of (otherwise) useful work or partially destruction of availability and this concept is useful in order to estimate the amount of energy that is converted into lower grade energy that can no longer find any meaningful utilization (has been turned into unavailable energy) to enable the machineries to work, such as, arotating shaft that suffers friction against the bearings. The work supplied (or produced) to overcome friction is then transformed into low-grade heat that cannot be used for further useful effects.

Although the practical applications explained are quite simple, however, the resulting calculations may proof to be very useful for operations because the collected data can be used, together with other *relevant measurements, to conduct condition monitoring of the devices and as a tool for energy audit to enhance efficiency in energy consumption by any one of those devices. In addition, the health of the equipment can be closely observed by taking note the change of relevant Entropy Number.*

CONCLUSIONS

The increase of entropy to operate work-related devices is important in assessing the total entropy increase of a Rankine cycle. In this paper, the increase of entropy formulations for pump and steam turbine as a function of their respective total efficiency have been derived and presented. An interesting conclusion is that entropy-increase relations for pump and turbine are showing similar functional form with the difference solely lies in their respective characteristic as work-related devices. Specifically, the difference stems from whether it is a work-consuming or a work-producing device. Also, this paper introduces two dimensionless number tentatively called Pump Entropy Number and Turbine Entropy Number that give dimensionless entropy increases which, in turn, can be used to measure the extend of useful work destruction as the result of inefficiencies of the two devices. Lastly, a convenient way to measure such destructions is given.

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